

On Non-Abelian T-Duality and new $\mathcal{N} = 1$ backgrounds

Georgios Itsios^{1,3*}, Carlos Núñez^{2†},
Konstadinos Sfetsos^{3,1‡} and Daniel C. Thompson^{4§}

¹Department of Engineering Sciences, University of Patras,
26110 Patras, Greece

²Swansea University, School of Physical Sciences,
Singleton Park, Swansea, SA2 8PP, UK

³Department of Mathematics, University of Surrey,
Guildford GU2 7XH, UK

⁴Theoretische Natuurkunde, Vrije Universiteit Brussel, and
The International Solvay Institutes
Pleinlaan 2, B-1050, Brussels, Belgium

Abstract

We study the action of non-Abelian T-duality in the context of $\mathcal{N} = 1$ geometries with well understood field theory duals. In the conformal case this gives rise to a new solution that contains an $AdS_5 \times S^2$ piece. In the case of non-conformal geometries we obtain a new background in massive IIA supergravity that presents similar behaviour to the cascade of Seiberg dualities. Some physical observables are discussed.

*gitsios@upatras.gr

†c.nunez@swansea.ac.uk

‡k.sfetsos@surrey.ac.uk

§dthompson@tena4.vub.ac.be

1 Introduction

T-duality, which in its simplest form states an equivalence between strings propagating on a circle of radius R and those on a circle of inverse radius α'/R , is a cornerstone of the web of dualities that exist within string theory and M-theory. A natural question to ask is whether T-duality may be generalised beyond the case of circular dimensions with $U(1)$ isometries to strings whose target space contains non-Abelian isometry groups. In a pioneering work on the subject [1] explains how to generalise the procedure introduced by Buscher (for Abelian T-duality) in [2]. Indeed, the process of gauging isometries, introducing Lagrange multipliers to enforce a flat connection and integrating out the gauge fields to produce a dual model, was extended to the case of non-Abelian isometries. Other important foundational work on the subject includes [3]-[6].

Beyond these initial breakthroughs two main difficulties emerged. Firstly it seemed rather hard to obtain "interesting" dual backgrounds in this manner and secondly the status of such non-abelian duality transformations as full symmetries of string (genus) perturbation theory is questionable. Nonetheless, it is reasonable to consider the role of non-abelian T-duality as a solution generating symmetry of the low energy effective action of string theory, i.e. supergravity. It is of particular interest to address this question in the context of the AdS/CFT correspondence [7].

A technical challenge that needed to be addressed, in light of the AdS/CFT correspondence, was to understand non-abelian T-duality in supergravity backgrounds with Ramond-Ramond fluxes. This was first achieved in [8] and has been extended in a number of recent works in [9] and [10]. A brief review of elementary aspects of non-Abelian T-duality can be found in [11]. Recently, a supersymmetric solution of Type IIB containing an AdS_6 factor was constructed using non-Abelian T-duality in [12].

In this letter, motivated by the AdS/CFT correspondence [7], we shall describe the utility and application of non-Abelian T-duality to Type II supergravity backgrounds with $\mathcal{N} = 1$ supersymmetry. We will find that (up to subtleties to be discussed) backgrounds of the form presented in [13] are found, starting from trademark solutions in Type IIB and non-abelian T-dualising them. In particular, we will present two *new* solutions. One of the form $AdS_5 \times S^2 \times M_4$ in eleven-dimensional Supergravity and another in Massive IIA Supergravity that may be thought as the 'cascading' version of the first. Some subtle points will be discussed, but we leave a detailed study of the properties of these geometries for [14].

2 The Technique of Non-Abelian Duality

In this letter we consider Type II backgrounds that have a freely acting $SU(2)$ symmetry such that metric may be decomposed as

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + 2G_{\mu i}(x)dx^\mu L^i + g_{ij}(x)L^i L^j, \quad (2.1)$$

where $\mu = 1, 2, \dots, 7$ and L^i are the left invariant Maurer–Cartan forms $L^i = -i\text{Tr}(g^{-1}dg)$. We also assume a similar ansatz for all the other fields.

The non-linear sigma model corresponding to this background is

$$S = \int d^2\sigma Q_{\mu\nu} \partial_+ X^\mu \partial_- X^\nu + Q_{\mu i} \partial_+ X^\mu L_-^i + Q_{i\mu} L_+^i \partial_- X^\mu + E_{ij} L_+^i L_-^j, \quad (2.2)$$

where

$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q_{\mu i} = G_{\mu i} + B_{\mu i}, \quad Q_{i\mu} = G_{i\mu} + B_{i\mu}, \quad E_{ij} = g_{ij} + b_{ij}. \quad (2.3)$$

To obtain the dual sigma model one first gauges the isometry by making the replacement

$$\partial_\pm g \rightarrow D_\pm g = \partial_\pm g - A_\pm g, \quad (2.4)$$

in the pulled-back Maurer–Cartan forms. In addition, a Lagrange multiplier term $-i\text{Tr}(vF_{+-})$ is added to enforce a flat connection.

After integrating this Lagrange multiplier term by parts, one can solve for the gauge fields to obtain the T-dual model. The final step of the process is to gauge fix the redundancy, for instance, by setting $g = \mathbb{1}$. In this way one obtains the Lagrangian,

$$\tilde{S} = \int d^2\sigma Q_{\mu\nu} \partial_+ X^\mu \partial_- X^\nu + (\partial_+ v_i + \partial_+ X^\mu Q_{\mu i})(E_{ij} + f_{ij}{}^k v_k)^{-1} (\partial_- v_j - Q_{j\mu} \partial_- X^\mu), \quad (2.5)$$

from which the T-dual metric and B-field can be ascertained. As with Abelian T-duality the dilaton receives a shift from performing the above manipulations in a path integral.

In principle, one ought to repeat the above process in a formalism that caters for the full background including RR fluxes. However the correct transformation rules for the RR fields may be obtained with the following recipe (which can be motivated by for instance considering the pure spinor superstring as in [15] for the related case of fermionic T-duality). One observes that after T-duality, left and right movers naturally couple to two different sets of vielbeins for the dual geometry. Since these two sets of frame fields describe the same metric they

are related by a Lorentz transformation which we denote by Λ . This Lorentz transformation induces an action on spinors defined by the invariance property of gamma matrices:

$$\Omega^{-1}\Gamma^a\Omega = \Lambda^a_b\Gamma^b. \quad (2.6)$$

To find the dual RR fluxes one simply acts by multiplication from the right with this Ω on the RR bispinor (or equivalently Clifford multiplication of the RR poly form). More explicitly, the T-dual fluxes \hat{P} are given by

$$\hat{P} = P \cdot \Omega^{-1}, \quad (2.7)$$

where

$$\text{IIB} : P = \frac{e^\Phi}{2} \sum_{n=0}^4 \hat{F}_{2n+1}, \quad \text{IIA} : \hat{P} = \frac{e^{\hat{\Phi}}}{2} \sum_{n=0}^5 \hat{F}_{2n}. \quad (2.8)$$

The chirality of the theory is preserved/switched when the isometry group dualised has even/odd dimension respectively. Full details and general expressions for the dual geometry, including alternate gauge fixing choices, for the dual geometry will be reported in the forthcoming publication [14].

3 The conformal case: T-dual of the Klebanov–Witten background

In [16] the system of D3-branes at the tip of the conifold was studied. The gauge theory on the branes is an $\mathcal{N} = 1$ superconformal field theory with product gauge group $SU(N) \times SU(N)$ and bifundamental matter fields. This gauge theory is dual to the Type IIB string theory on $AdS^5 \times T^{(1,1)}$ with N units of RR flux on the $T^{(1,1)}$. The geometry and the 5-form self-dual flux form, are given by

$$\begin{aligned} ds^2 &= \frac{r^2}{L^2} dx_{1,3}^2 + \frac{L^2}{r^2} dr^2 + L^2 ds_{T^{1,1}}^2, \\ F_{(5)} &= \frac{4}{g_s L} (\text{Vol}(AdS_5) - L^5 \text{Vol}(T_{1,1})) . \end{aligned} \quad (3.1)$$

Here $T^{(1,1)}$ is the homogenous space $(SU(2) \times SU(2))/U(1)$ with the diagonal embedding of the $U(1)$. It has an Einstein metric with $R_{ij} = 4g_{ij}$ given by

$$ds_{T^{(1,1)}}^2 = \lambda_1^2 (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \lambda_2^2 (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) + \lambda^2 (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2. \quad (3.2)$$

with $\lambda_1^2 = \lambda_2^2 = \frac{1}{6}$ and $\lambda^2 = \frac{1}{9}$. In these conventions $L^4 = \frac{27}{4} g_s N \pi$ ensures that the charge $\int_{T^{1,1}} F_5 = 16\pi^4 N$ is correctly quantised for integer N .

We perform a dualisation with respect to the $SU(2)$ isometry that acts on the θ_2, ϕ_2, ψ coordinates. The result of the dualisation procedure¹ is a target space with NS fields given by

$$\begin{aligned}
\widehat{ds}^2 &= ds_{\text{AdS}_5}^2 + \lambda_1^2(d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{\lambda_2^2 \lambda^2}{\Delta} x_1^2 \sigma_3^2 \\
&\quad + \frac{1}{\Delta} \left((x_1^2 + \lambda^2 \lambda_2^2) dx_1^2 + (x_2^2 + \lambda_2^4) dx_2^2 + 2x_1 x_2 dx_1 dx_2 \right) , \\
\widehat{B} &= -\frac{\lambda^2}{\Delta} \left[x_1 x_2 dx_1 + (x_2^2 + \lambda_2^4) dx_2 \right] \wedge \sigma_3 , \\
e^{-2\widehat{\Phi}} &= \frac{8}{g_s^2} \Delta ,
\end{aligned} \tag{3.3}$$

where $\sigma_3 = d\psi + \cos \theta_1 d\phi_1$ and

$$\Delta \equiv \lambda_2^2 x_1^2 + \lambda^2 (x_2^2 + \lambda_2^4) . \tag{3.4}$$

This geometry is regular and the dilaton never blows up. For a fixed value of (x_1, x_2) the remaining directions give a squashed three sphere. The metric evidently has a $SU(2) \times U(1)_\psi$ isometry.

Following the procedure outlined above—see [14] for details— one can determine the RR fluxes that support this geometry to be

$$\begin{aligned}
\widehat{F}_2 &= \frac{8\sqrt{2}}{g_s} \lambda_1^4 \lambda \sin \theta_1 d\phi_1 \wedge d\theta_1 , \\
\widehat{F}_4 &= -\frac{8\sqrt{2}}{g_s} \lambda_1^2 \lambda_2^2 \lambda \frac{x_1}{\Delta} \sin \theta_1 d\phi_1 \wedge d\theta_1 \wedge d\psi \wedge (\lambda_2^2 x_1 dx_2 - \lambda^2 x_2 dx_1) .
\end{aligned} \tag{3.5}$$

This background enjoys $\mathcal{N} = 1$ supersymmetry and its explicit Killing spinors can be determined by the expression

$$\hat{\eta} = \Omega \cdot \eta \tag{3.6}$$

where η are the Killing spinors of the Klebanov–Witten background and the Ω matrix defined in (2.6) has the form

$$\Omega = \frac{1}{\sqrt{\Delta}} \Gamma_{11} \left(-\lambda \lambda_2^2 \Gamma_{123} + \lambda_2 x_1 \Gamma_1 + \lambda x_2 \Gamma_3 \right) . \tag{3.7}$$

One could anticipate this result since the $U(1)_R$ symmetry commutes with the $SU(2)$ used in the T-duality. Hence one expects the corresponding isometry to be preserved after dualisation. Indeed one can explicitly verify that the Killing spinors of the Klebanov–Witten

¹To obtain this we actually chose to fix the gauge symmetry by taking $\theta_2 = \phi_2 = v_2 = 0$, rather than simply $g = 1$ since it makes manifest the residual isometries. Additionally for aesthetic reasons we rename $v_1 = 2x_1$ and $v_3 = 2x_2$ and set $L = 1$.

background have vanishing spinor-Lorentz-Lie derivative along the three Killing vectors that generate the $SU(2)$ isometry.

It is interesting to ask what are the charges of extended objects in this background. Because of the non zero NS two-form, the Chern–Simons terms play an important role and in general, the notion of charge that is quantised is the Page charge. There is a natural two cycle in the geometry, $\Sigma_2 = \{\theta_1, \phi_1\}$, over which the D6 charge can be measured by integrating \hat{F}_2 . One finds that the D3 charge has been converted to D6 charge after dualisation. Since there is no natural four-cycle in the geometry there are no other natural D-brane charges; the activation of \hat{F}_4 is required to solve the supergravity equations of motion.

A natural question to ask is, what is the field theory dual to this geometry. As a first step one might wish to calculate the central charge, which essentially is done by measuring the volume of the internal manifold. A remarkable feature of non-Abelian T-duality is that this volume is conserved in the following sense;

$$e^{-2\Phi} \sqrt{\det g} \Delta_{F.P.} = e^{-2\hat{\Phi}} \sqrt{\det \hat{g}}, \quad (3.8)$$

where $\Delta_{F.P.}$ is the Fadeev–Popov determinant that arises from gauge fixing to obtain the dual sigma model. That is to say all of the complexity of the metric cancels against that of the dilaton leaving a rather simple result. As we will see in the non-conformal case this implies that the central charges match up to an RG scale independent multiplicative constant. Such a relation was first shown for gauged WZW models in [17], but it is valid the context of non-Abelian duality as well.

The lift to eleven dimensions (along the circle with coordinate $x_\#$) of the geometry we found in eq.(3.3), is given by

$$\begin{aligned} ds^2 = & \Delta^{1/3} (ds_{AdS_5}^2 + \lambda_1^2 (\sigma_1^2 + \sigma_2^2)) + \Delta^{-2/3} [(x_1^2 + \lambda^2 \lambda_1^2) dx_1^2 \\ & + (x_2^2 + \lambda_1^4) dx_2^2 + 2x_1 x_2 dx_1 dx_2 + \lambda^2 \lambda_1^2 x_1^2 \sigma_3^2 + \left(dx_\# + \frac{\sigma_3}{27}\right)^2], \end{aligned} \quad (3.9)$$

where Δ is given in (3.4). The four-form flux field is given by

$$F_4 = d(C_3 + B \wedge dx_\#) = \frac{1}{27} dx_2 \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3 + H \wedge dx_\#, \quad (3.10)$$

where $H = dB$ is computed using the expression for B in (3.3).

Recently, a class of $\mathcal{N} = 1$ (generically non-Lagrangian) SCFT's found as the IR fixed point of the dynamics of M5-branes wrapped on a genus g surface Σ_g was engineered [13, 18].

These field theories enjoy not only a $U(1)_R$ global symmetry but also an additional $U(1)$ global symmetry. Moreover in [13, 18] the geometrical dual to these solutions was given. Rather remarkably our solution fits in this ansatz for the case of genus zero (the sphere). This is an intriguing connection and certainly hints towards a field theoretic interpretation however two caveats must be made; firstly that the field theories of [13, 18] are less well understood in general for the case of genus zero and secondly that even within the solutions presented in [13, 18], ours is special. Our solution, whilst a solution of eleven-dimensional supergravity, appears not to be a fixed point of the BPS equations of the corresponding seven-dimensional gauged supergravity studied in [13, 18, 19]. Moreover our solution corresponds to a particular limiting value of the parameters that classify the eleven-dimensional solutions in [13, 18].

Let us remark further on some similarity with the situation considered in [8] where the same $SU(2)$ non-Abelian dualisation was performed on $AdS_5 \times S^5$. In that case the resultant geometry corresponded to a limit of the Gaiotto-Maldacena geometries [20], dual to $\mathcal{N} = 2$ SCFTs presented in [21]. Although there supersymmetry was halved by the dualisation whereas here it is preserved, what we have here can be viewed as an $\mathcal{N} = 1$ parallel to [8]. Indeed, the theories considered in [13, 18] are really $\mathcal{N} = 1$ cousins of the Gaiotto $\mathcal{N} = 2$ theories and can be obtained by integrating out some $\mathcal{N} = 1$ scalars contained in $\mathcal{N} = 2$ vector multiplets. An interesting question to ask is if one can use a similar procedure to dualise the entire flow between $AdS_5 \times S^5/\mathbb{Z}_2$ and $AdS_5 \times T^{1,1}$ geometries to provide a gravity description of the flow between the $\mathcal{N} = 2$ SCFTs in [21] and the $\mathcal{N} = 1$ in [13, 18].

4 The non-conformal case: T-dual of the Klebanov-Tseytlin solution

Let us now turn our attention to non-conformal backgrounds obtained by placing M fractional D3-branes i.e. D5-branes wrapping a contractible two cycle of $T^{(1,1)}$ as in [22, 23]. This modifies the field theory to be $SU(N) \times SU(N + M)$, hence no longer conformal. In fact this theory has rich RG dynamics undergoing a sequence of Seiberg dualities to lower rank gauge groups as one proceeds to the IR. In the IR, strong coupling dynamics takes hold giving rise to spontaneous Z_{2M} -symmetry breaking, confinement and other non-perturbative effects.

Let us here discuss the case of Klebanov-Tseytlin (KT) [22], details of the full Klebanov Strassler geometry [23] and related $\mathcal{N} = 1$ backgrounds [24] will appear in [14].

The geometry is given [22] by²

$$ds^2 = h^{-1/2}(r)dx_{1,3}^2 + h^{1/2}(r) (dr^2 + r^2 ds_{T^{1,1}}^2) \quad (4.1)$$

where the warping function displays the characteristic logarithmically running

$$h = b_0 + \frac{p^2}{4r^4} \ln(r/r_*) . \quad (4.2)$$

This is supported by fluxes

$$B_2 = -T(r)\omega_2 , \quad F_3 = -Pe^\psi \wedge \omega_2 , \quad F_5 = (1 + *)K(r)\text{vol}(T^{1,1}) \quad (4.3)$$

where the forms e^ψ and ω_2 are the conventional ones defined on $T^{1,1}$ and may be found explicitly in [22].

In fact, this is a particular solution of a class of KT-geometries characterised by a set of functions obeying some BPS equations. Although in this letter we only consider this special solution it can be shown that the whole ansatz can be non-Abelian T-dualised and solves the supergravity equations of motion subject to the same BPS equations.

Again we perform the non-Abelian duality with respect to an $SU(2)$ isometry and find a dual geometry given by

$$\begin{aligned} \hat{ds}^2 &= h^{-1/2}(r)dx_{1,3}^2 + h^{1/2}(r) \left(dr^2 + \frac{1}{6}(d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) \right) + \hat{ds}_3^2 \\ \hat{ds}_3^2 &= \frac{1}{2r^2 \Delta h^{1/2}(r)} \left(12r^4 h(r) v_2^2 v_3^2 + 12(r^4 h(r) + 27v_2^2) dv_2^2 + 9(2r^4 h(r) + \mathcal{V}^2) dv_3^2 + 108\mathcal{V} v_2 dv_2 dv_3 \right) . \end{aligned} \quad (4.4)$$

with

$$\Delta = 2r^4 h(r) + \mathcal{V}^2 + 54v_2^2 , \quad \mathcal{V} = 6v_3 - T(r) . \quad (4.5)$$

This geometry is supported in the NS sector by both a dilaton and a two-form,

$$\begin{aligned} \hat{B} &= -\frac{T(r)}{6\sqrt{2}} \sin \theta_1 d\theta_1 d\phi_1 + \frac{3\sqrt{2}}{\Delta} \mathcal{V} v_2 \sigma_3 \wedge dv_2 + \frac{1}{\sqrt{2}\Delta} (2r^4 h(r) + \mathcal{V}^2) \sigma_3 \wedge dv_3 , \\ e^{-2\hat{\Phi}} &= \frac{1}{81} r^2 h(r)^{\frac{1}{2}} \Delta . \end{aligned} \quad (4.6)$$

In the RR sector we find

$$\hat{F}_0 = \frac{P}{9} , \quad \hat{F}_2 = \frac{2K(r) - P\mathcal{V}}{54\sqrt{2}} \sin \theta_1 d\theta_1 \wedge d\phi_1 + \frac{\sqrt{2}P\mathcal{V}v_2}{3\Delta} \sigma_3 \wedge dv_2 - \frac{3\sqrt{2}Pv_2^2}{\Delta} \sigma_3 \wedge dv_3 ,$$

²The dilaton is constant and we have set it equal to 1 so that there is no difference between string and Einstein frame.

$$\hat{F}_4 = \frac{v_2}{18\Delta} \sin \theta_1 d\theta_1 \wedge d\phi_1 \wedge d\psi \wedge \left(-9(2K(r) - P\mathcal{V})v_2 dv_3 + 2(Pr^4 h(r) + \mathcal{V}K(r) + 27Pv_2^2)dv_2 \right). \quad (4.7)$$

The metric has some similarities with the case of the dualised KW theory which is to be expected. However, the RR sector reveals a striking difference; this is a solution of *massive* type IIA supergravity with the Roman's mass obeying a natural quantisation given by P which measured the number of fractional branes prior to dualisation. Indeed the Page charges of this solution,

$$Q_{Page,D6} = \frac{1}{\sqrt{2}\pi^2} \int_{\theta\phi} \hat{F}_2 - \hat{F}_0 \hat{B} = \frac{2Q}{27\pi}, \quad Q_{Page,D8} = \sqrt{2} \int \hat{F}_0 = \frac{\sqrt{2}P}{9}, \quad (4.8)$$

show that what was D3 charge has become D6 charge and what was D5 charge has become D8 charge (a result which chimes well with the naive view of performing three T-dualities). There is no obvious cycle for D4 charge to be measure over. Before dualisation the duality cascade could be seen by studying the charges. Indeed, two equivalent views [25] of this are the changes seen in the D3 Maxwell charge as the radial coordinate is varied or the jumps in the Page charge induced by large gauge transformations such that $\frac{1}{4\pi^2} \int B_2$ changes by an integer. Indeed one finds an analogous behaviour in the charges after dualisation again suggestive of some field theory cascade interpretation. One subtlety is that a change of M units in the charges of the KT geometry becomes a change of $2M$ units in the transformed geometry. Giving a complete field theory description of this set up remains an interesting problem, but is beyond the scope of this letter; see [14].

As we indicated earlier the invariance of the stringy volume of the internal manifold to be dualised, has strong implications for the central charge. In particular if we calculate the central charge following the procedure explained in [26] (modified slightly to accommodate a dilaton that may depend on the internal dimensions) one finds in the original geometry of eq.(4.1),

$$c = \frac{2\pi^3}{27A'(r)^3} \quad (4.9)$$

and after dualisation

$$\hat{c} = \frac{\sqrt{2}\pi^2}{27A'(r)^3} \times \mathcal{I} \quad (4.10)$$

where $A(r)$ is defined in [26] and given by $e^{2A(r)} = h(r)^{\frac{1}{3}} r^{\frac{10}{3}}$. One sees that the two agree up to a single RG scale invariant constant that is set by the periodicities of the dual coordinates. More precisely this constant \mathcal{I} is determined entirely by the rather subtle global properties of the T-dual coordinates, in this case we have $\mathcal{I} = \int dv_3 \int dv_2 v_2$. An important question

for further study is to better understand such global issues, either via the sigma model or via space time considerations.

5 Discussion

Developing the field theory duals corresponding to these geometries represents the most obvious open problem. One approach is to consider various D brane probes and 'define' the field theory via its observables, calculated in a smooth background (with all IR effects taken into account). This analysis is already underway and shall be reported in [14]. Nevertheless, a more canonical approach, based on a careful field theory analysis following the lead of [21], [27] may be in order.

We believe that as well as developing the particular cases studied above this work opens up many possible new lines of research. Firstly a more general classification of massive type IIA backgrounds that display similar signatures of cascade would be highly desirable. Equally one could hope to use the techniques outlined above to find new and interesting classes of backgrounds. Indeed in this work and other recent studies, it seems that we have only just started seeing the utility of these duality transformations. In principle whenever a space time admits a non-abelian isometry these techniques might be applicable. There are, of course, many such examples and we hope that further study will prove fruitful.

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